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SUMMATION OF THE EFFECT OF VARYING TEMPERATURES ON THE METABOLISM OF A BIOLOGICAL SYSTEM

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SUMMARY

Several relationships obtained during investigations of methods for determining the loss of green-life of banana fruit are presented.

I. INTRODUCTION

Green-life of climacteric-type fruit has been defined by Peacock and Blake (1970) as the time that elapses until the onset of the respiratory climacteric rise under defined conditions.

The development of a method for determining the loss of green-life of banana fruit under conditions of varying temperature has yielded several relationships which may be of use to other biologists, as many biological systems exhibit characteristics which, like green-life (Peacock and Blake 1971) vary exponentially with temperature.

$$\text{Since } \ln G = mT + c \quad \ldots\ldots\ldots\ldots \quad (1)$$

(G = green-life, T = temperature and m and c are constants), eliminating the constant c we find:

$$G_{T_0} = G_T e^{m(T_0 - T)}$$

where G_{T_0}, G_T are the green-lives at temperatures T_0 and T respectively. This enables us to convert the green-life at temperature T to its equivalent at some arbitrary but standard temperature, T_0. Also, any portion of the green-life (g_T) at temperature T may also be converted to an equivalent portion (g_{T_0}) at temperature T_0. Thus

$$g_{T_0} = g_T e^{m(T_0 - T)}$$

II. RELATIONSHIPS

(a) Discrete Case

Suppose that during an interval of time (t_1,t_2) the temperature T assumes n discrete values $T_1,\ldots, T_n$ during n sub-intervals of time $\delta t_1,\ldots, \delta t_n$. Then during the time interval δt_i, the temperature is T_i and

$$\delta g_{T_o} = \delta g_{T_i}\, e^{m(T_o - T_i)}$$

However, δt_i is δg_{T_i}, so that

$$\delta g_{T_o} = \delta t_i e^{m(T_o - T_i)}$$

Summing, $$g_{T_o} = \sum_{i=1}^{n} \delta t_i e^{m(T_o - T_i)}\ , \ldots\ldots (2)$$

where g_{T_o} is the expiration of green-life at temperature T_o corresponding to the sum of the expirations of the green-lives g_{T_i}, $i = 1, 2, 3, \ldots n$ at the temperature to which the fruit is subjected.

(b) Continuous Case

Proceeding to the limit as $\delta t_i \to O$ and $n \to \infty$, equation (2) may be written as:

$$g_{T_o} = \int_{t_1}^{t_2} e^{m(T_o - f(t))}\, dt$$

$$= e^{mT_o} \int_{t_1}^{t^2} e^{-mf(t)}\, dt \ldots\ldots (3)$$

where f (t) is the temperature T at time t.

(c) Temperatures Varying According to Newton's Law of Cooling

When $f(t) = a + b\, e^{kt}$ (a = final temperature, $a + b$ = initial temperature), equation (3) becomes:

$$g_{T_o} = e^{m(T_o - a)} \int_{t1}^{t^2} e^{-mbe^{kt}}\, dt.$$

Taking the initial time t, as zero, the substitution $u = -mbe^{kt}$ reduces the integral on the right hand side to the form

$$\int \frac{e^u\, du}{uk} = \int \frac{1}{uk} \left(1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \ldots \frac{u^p}{p!} + \ldots\right) du$$

$$= \frac{1}{k} \left[\ln u + u + \frac{u^2}{2.2!} + \ldots + \frac{u^p}{p.p!} + \ldots \right]_{-mb}^{-mbe^{kt}}$$

and $$g_{T_o} = \frac{e^{m(T_o - a)}}{k} \Big[\ln(-mbe^{kt}) - \ln(-mb) + (-mbe^{kt}) - (-mb)$$

$$+ \frac{(-mbe^{kt})^2 - (-mb)^2}{2.2!} + \ldots + \frac{(-mbe^{kt})^p - (-mb)^p}{p.p!} + \ldots \Big]$$

$$\doteq\approx \frac{e^{m(T_o - a)}}{k} \left[kt + \sum_{j=1}^{p} d_j (e^{jkt} - 1) \right]$$

where $d_j = \dfrac{(-mb)^j}{j.j!}$

which can be evaluated approximately by taking the first p terms of the infinite series.

(d) Temperatures Varying Sinusoidally

When $f(t) = a + b \sin 2\pi t$

(a = mean temperature of cycle, $b = \frac{1}{2}$ temperature range).

Equation 3 becomes:

$$g_{T_o} = \int_{t_1}^{t_2} e^{m(T_o - a - b \sin 2\pi t)} \, dt \qquad = e^{m(T_o - a)} \int_{t_1}^{t_2} e^{-m b \sin 2\pi t} \, dt$$

$$= e^{m(T_o - a)} \int_{t_1}^{t_2} \left[1 + (-mb \sin 2\pi t) + \frac{(-mb \sin 2\pi t)^2}{2!} + \ldots \right.$$

$$\left. + \frac{(-mb \sin 2\pi t)^p}{p!} + \ldots \right] dt$$

$$\approx e^{m(T_o - a)} \sum_{j=o}^{p} d_j \int_{t_1}^{t_2} (\sin 2\pi t)^j \, dt, \qquad \text{where } d_j = \frac{(-mb)^j}{j!}$$

This is evaluated approximately using standard reduction formulae for the first p terms of the infinite series.

Williams (1969), assuming metabolic rates respond exponentially to changes in temperature, has published useful tables of mean effective temperatures for biological systems under sinusoidally fluctuating temperatures. These are quoted for biological systems having Q_{10} values between 1 and 3, and have been calculated for various amplitudes of the sine curves. In a system where whole time cycles are being considered it is simpler to use the mean effective temperature for a system rather than evaluate the above series. From the equation $\ln R = mT + C$ (where R is metabolic rate and T is temperature) it can be shown that $Q_{10} = e^{10m}$. Hence, provided m is known, the Q_{10} value for a system can be simply calculated. It should be noted that although the mean effective temperatures quoted by Williams are valid for cycles of any length, their use for a portion of a cycle is not valid and the method derived in this paper should be used.

If needed, other substitutions (where $T = f(t)$ is simple) may be made in Equation 3. Where the relationship is not mathematically simple, stepwise summation of discrete intervals using Equation 2 may be a more suitable procedure.

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